Enrollment No: ______________________ **Exam Seat No: ______________________**

C. U. SHAH UNIVERSITY

Summer Examination-2020

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT2 **Branch: B.Tech (All)**

Semester : 1 **Date : 26/02/2020** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) nth derivative of $y=e^{-x}$ is

(A) $-e^{-x}$ (B) $(-1)^n e^{-x}$ (C) $(-1)^{n+1} e^{-x}$ (D) none of these

b) If $y=\log(5-2x)$, then y_n equal to

(A) $\frac{(-1)^n n!(-2)^n}{(5-2x)^{n+1}}$ (B) $\frac{(-1)^n n!(-2)^n}{(5-2x)^n}$ (C) $\frac{(-1)^{n-1}(n-1)!(-2)^n}{(5-2x)^n}$

(D) $\frac{(-1)^n n!(-2)^n}{(5-2x)^{n-1}}$

c) If $y=\sin^{-1}x$, then x equal to

(A) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-.....$ (B) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}+.....$ (C) $1-y+\frac{y^2}{2!}-\frac{y^3}{3!}+.....$

(D) $y-\frac{y^3}{3!}+\frac{y^5}{5!}-.....$

d) If $y=x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+.....$ then x equal to

(A) $y-\frac{1}{2}y^2+\frac{1}{3}y^3-\frac{1}{4}y^4+.....$ (B) $y-\frac{1}{2!}y^2+\frac{1}{3!}y^3-\frac{1}{4!}y^4+.....$

(C) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}+.....$ (D) none of these

e) $\lim_{x\to 0}\frac{5^x-3^x}{x}=$ ______

(A) 2 (B) $\log 2$ (C) $\log 15$ (D) $\log\left(\frac{5}{3}\right)$

f) $\lim_{x\to\infty} x^n e^{-ax}$ (n being a positive integer and $a>0$) = ______

(A) – 1 (B) 0 (C) 1 (D) None of these

g) If $x = r\cos\theta,\ y = r\sin\theta,$ then $\frac{\partial r}{\partial x}$ is equal to

(A) $\sec\theta$ (B) $\sin\theta$ (C) $\cos\theta$ (D) $\operatorname{cosec}\theta$

h) If $u = f\left(\frac{x}{y}\right)$ then

(A) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$ (B) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$ (C) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$

(D) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

i) If $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)}$ is equal to

(A) 1 (B) – 1 (C) zero (D) none of these

j) Conditions for $f(x,y)$ to be maximum are

(A) $f_x = 0 = f_y,\ rt < s^2,\ r < 0$ (B) $f_x = 0 = f_y,\ rt > s^2,\ r < 0$

(C) $f_x = 0 = f_y,\ rt > s^2,\ r > 0$ (D) $f_x = 0 = f_y,\ rt = s^2,\ r > 0$

k) If $1,\ \alpha_1,\ \alpha_2,\ \ldots,\ \alpha_{n-1}$ are the n roots of unity, then $(1+\alpha_1)(1+\alpha_2)\ldots(1+\alpha_{n-1})$ is equal to

(A) $n-1$ (B) n (C) – 1 (D) none of these

l) If $y = \cos\theta + i\sin\theta$, then the value of $y + \frac{1}{y}$ is

(A) $2\cos\theta$ (B) $2\sin\theta$ (C) $2\operatorname{cosec}\theta$ (D) $2\tan\theta$

m) If every minor of order r of a matrix A is zero, then rank of A is

(A) greater than r (B) equal to r (C) less than or equal to r

(D) less than r

n) An eigenvalue of a square matrix A is $\lambda = 0$. Then

(A) $|A| \neq 0$ (B) A is symmetric (C) A is singular

(D) A is skew-symmetric

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) If $y = \frac{x^4}{(x-1)(x-2)}$ then find y_n. **(5)**

b) Prove that $(1+x)^x = 1 + x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - \ldots$ **(5)**

c) If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$ then show that **(4)**

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}.$$

Q-3 **Attempt all questions** **(14)**

	a)	If $\cos^{-1}\left(\frac{y}{b}\right)=\log\left(\frac{x}{n}\right)^n$ then prove that $x^2y_{n+2}+(2n+1)xy_{n+1}+2n^2y_n=0$.	(5)
	b)	Expand $\tan^{-1}x$ up to the first four terms by Maclaurin's series.	(5)
	c)	Evaluate: $\lim_{x\to 0}\frac{e^x+e^{-x}-x^2-2}{\sin^2 x-x^2}$	(4)
Q-4		**Attempt all questions**	**(14)**
	a)	Evaluate: $\lim_{x\to 0}\left(\frac{1}{x}\right)^{1-\cos x}$	(5)
	b)	Find the approximate value of $\sqrt{27}\sqrt[3]{1021}$ using partial differentiation.	(5)
	c)	Expand $\log x$ in powers of $(x-2)$.	(4)
Q-5		**Attempt all questions**	**(14)**
	a)	If $u=\sec^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then prove that $x^2\frac{\partial^2u}{\partial x^2}+2xy\frac{\partial^2u}{\partial x\partial y}+y^2\frac{\partial^2u}{\partial y^2}=-\cot u\left(\cot^2 u+2\right)$.	(5)
	b)	Evaluate: $\lim_{x\to a}\log\left(2-\frac{x}{a}\right)\cot(x-a)$	(5)
	c)	If $y=\cos x\cos 2x\cos 3x$ then find y_n .	(4)
Q-6		**Attempt all questions**	**(14)**
	a)	The power consumed in an electric resistor is given by $P=\frac{E^2}{R}$ (in watts). If $E=200$ volts and $R=8$ ohms, by how much does the power change if E is decreased by 5 volts and R is decreased by 0,20 ohms?	(5)
	b)	Find the continued product of all the values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$.	(5)
	c)	Investigate for what values of λ and μ the equations $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$, have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.	(4)
Q-7		**Attempt all questions**	**(14)**
	a)	Find the eigenvalues and eigenvectors of matrix $A=\begin{bmatrix}-3&-7&-5\\2&4&3\\1&2&2\end{bmatrix}$.	(5)
	b)	Using De Moivre's theorem prove that $\cos 5\theta=5\cos\theta-20\cos^3\theta+16\cos^5\theta$	(5)
	c)	Prove that $\sec h^{-1}(\sin\theta)=\log\cot\frac{\theta}{2}$.	(4)
Q-8		**Attempt all questions**	**(14)**

a) Find the inverse of $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}$ by Gauss-Jordan reduction method. **(5)**

b) Find the fourth roots of unity and sketch them on the unit circle. **(5)**

c) Examine for linear dependence of vectors **(4)**

$(1,\ 2,\ -1,\ 0),\ (1,\ 3,\ 1,\ 2),\ (4,\ 2,\ 1,\ 0),\ (6,\ 1,\ 0,\ 1)$

and find a relation between them if dependent.